OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4722/01**

MATHEMATICS

Core Mathematics 2

WEDNESDAY 9 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

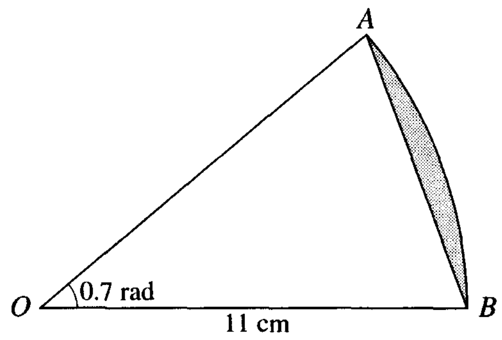
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

 [Turn over

1



The diagram shows a sector AOB of a circle with centre O and radius 11 cm. The angle AOB is 0.7 radians. Find the area of the segment shaded in the diagram. [4]

2 Use the trapezium rule, with 3 strips each of width 2, to estimate the value of

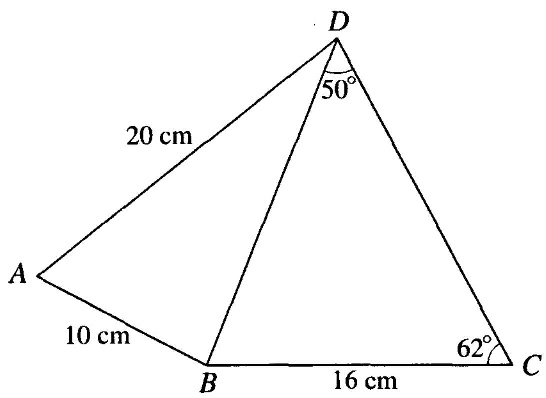
$$\int_1^7 \sqrt{x^2+3}\,dx. \qquad [4]$$

3 Express each of the following as a single logarithm:

(i) $\log_a 2 + \log_a 3$, [1]

(ii) $2\log_{10} x - 3\log_{10} y$. [3]

4



In the diagram, angle $BDC = 50°$ and angle $BCD = 62°$. It is given that $AB = 10$ cm, $AD = 20$ cm and $BC = 16$ cm.

(i) Find the length of BD. [2]

(ii) Find angle BAD. [3]

5 The gradient of a curve is given by $\frac{dy}{dx} = 12\sqrt{x}$. The curve passes through the point (4, 50). Find the equation of the curve. [6]

6 A sequence of terms u_1, u_2, u_3, ... is defined by

$$u_n = 2n + 5, \quad \text{for } n \geqslant 1.$$

(i) Write down the values of u_1, u_2 and u_3. [2]

(ii) State what type of sequence it is. [1]

(iii) Given that $\sum_{n=1}^{N} u_n = 2200$, find the value of N. [5]

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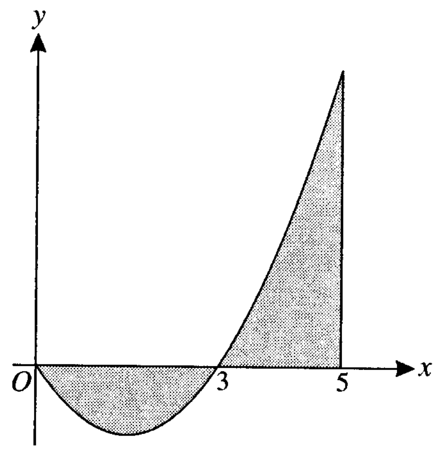

The diagram shows part of the curve $y = x^2 - 3x$ and the line $x = 5$.

(i) Explain why $\int_0^5 (x^2 - 3x)\,dx$ does not give the total area of the regions shaded in the diagram. [1]

(ii) Use integration to find the exact total area of the shaded regions. [7]

8 The first term of a geometric progression is 10 and the common ratio is 0.8.

(i) Find the fourth term. [2]

(ii) Find the sum of the first 20 terms, giving your answer correct to 3 significant figures. [2]

(iii) The sum of the first N terms is denoted by S_N, and the sum to infinity is denoted by S_∞.

Show that the inequality $S_\infty - S_N < 0.01$ can be written as

$$0.8^N < 0.0002,$$

and use logarithms to find the smallest possible value of N. [7]

9 **(i)**

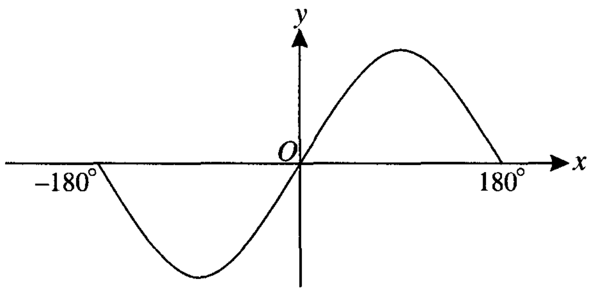

Fig. 1

Fig. 1 shows the curve $y = 2\sin x$ for values of x such that $-180° \leqslant x \leqslant 180°$. State the coordinates of the maximum and minimum points on this part of the curve. [2]

(ii)

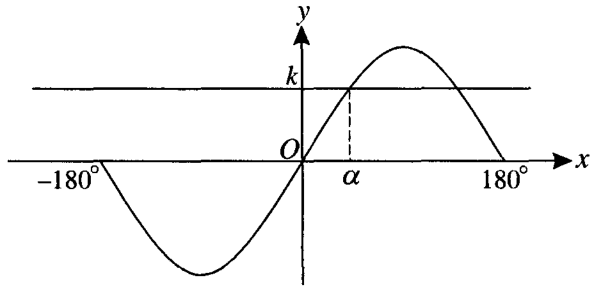

Fig. 2

Fig. 2 shows the curve $y = 2\sin x$ and the line $y = k$. The smallest positive solution of the equation $2\sin x = k$ is denoted by α. State, in terms of α, and in the range $-180° \leqslant x \leqslant 180°$,

(a) another solution of the equation $2\sin x = k$, [1]

(b) one solution of the equation $2\sin x = -k$. [1]

(iii) Find the x-coordinates of the points where the curve $y = 2\sin x$ intersects the curve $y = 2 - 3\cos^2 x$, for values of x such that $-180° \leqslant x \leqslant 180°$. [6]

10 **(i)** Find the binomial expansion of $(2x + 5)^4$, simplifying the terms. [4]

(ii) Hence show that $(2x + 5)^4 - (2x - 5)^4$ can be written as

$$320x^3 + kx,$$

where the value of the constant k is to be stated. [2]

(iii) Verify that $x = 2$ is a root of the equation

$$(2x + 5)^4 - (2x - 5)^4 = 3680x - 800,$$

and find the other possible values of x. [6]

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		Mark	Total	
1	area of sector = ½ x 11^2 x 0.7	M1		Attempt sector area using (½) $r^2\theta$
	= 42.35	A1		Obtain 42.35, or unsimplified equiv, soi
	area of triangle = ½ x 11^2 x sin0.7 = 38.98	M1		Attempt triangle area using ½absin$C$ or equiv, and
	hence area of segment = 42.35 – 38.98			subtract from attempt at sector
	= 3.37	A1	4	Obtain 3.37, or better
			$\boxed{4}$	
2	area $\approx \frac{1}{2}\times 2\times\left\{2+2\left(\sqrt{12}+\sqrt{28}\right)+\sqrt{52}\right\}$	M1		Attempt y-values at x = 1, 3, 5, 7 only
		M1		Correct trapezium rule, any h, for their y values to find area between x = 1 and x = 7
		M1		Correct h (soi) for their y values
	≈ 26.7	A1	4	Obtain 26.7 or better (correct working only)
			$\boxed{4}$	
3	**(i)** $\log_a 6$	B1	1	State $\log_a 6$ cwo
	(ii) $2\log_{10}x - 3\log_{10}y = \log_{10}x^2 - \log_{10}y^3$	M1*		Use $b \log a = \log a^b$ at least once
	$= \log_{10}\frac{x^2}{y^3}$	M1dep*		Use $\log a - \log b = \log {}^{a}/_{b}$
		A1	3	Obtain $\log_{10}\frac{x^2}{y^3}$ cwo
			$\boxed{4}$	
4	**(i)** $\frac{BD}{\sin 62} = \frac{16}{\sin 50}$	M1		Attempt to use correct sine rule in ΔBCD, or equiv.
	BD = 18.4 cm	A1	2	Obtain 18.4 cm
	(ii) $18.4^2 = 10^2 + 20^2 - 2 \times 10 \times 20 \times \cos\theta$	M1		Attempt to use correct cosine rule in ΔABD
	$\cos\theta = 0.3998$	M1		Attempt to rearrange equation to find cos BAD (from $a^2 = b^2 + c^2 \pm (2)bc\cos A$)
	$\theta = 66.4^0$	A1	3	Obtain 66.4^0
			$\boxed{5}$	
5	$\int 12x^{\frac{1}{2}}\,dx = 8x^{\frac{3}{2}}$	M1		Attempt to integrate
		A1√		Obtain correct, unsimplified, integral following their f(x)
		A1		Obtain $8x^{\frac{3}{2}}$, with or without + c
	$y = 8x^{\frac{3}{2}} + c \Rightarrow 50 = 8\times 4^{\frac{3}{2}} + c$	M1		Use (4, 50) to find c
	$\Rightarrow c = -14$	A1√		Obtain c = -14, following $kx^{\frac{3}{2}}$ only
	Hence $y = 8x^{\frac{3}{2}} - 14$	A1	6	State $y = 8x^{\frac{3}{2}} - 14$ aef, as long as single power of x
			$\boxed{6}$	

			Mark	Total	
6	**(i)**	$u_1 = 7$	B1		Correct u_1
		$u_2 = 9$, $u_3 = 11$	B1	**2**	Correct u_2 and u_3
	(ii)	Arithmetic Progression	B1	**1**	Any mention of arithmetic
	(iii)	$\frac{1}{2} N (14 + (N - 1) \times 2) = 2200$	B1		Correct interpretation of sigma notation
			M1		Attempt sum of AP, and equate to 2200
		$N^2 + 6N - 2200 = 0$	A1		Correct (unsimplified) equation
		$(N - 44)(N + 50) = 0$	M1		Attempt to solve 3 term quadratic in N
		hence $N = 44$	A1	**5**	Obtain $N = 44$ only ($N = 44$ www is full marks)
				$\boxed{8}$	
7	**(i)**	Some of the area is below the x-axis	B1	**1**	Refer to area / curve below x-axis or 'negative area'…
	(ii)		M1		Attempt integration with any one term correct
			A1		Obtain $\frac{1}{3}x^3 - \frac{3}{2}x^2$
		$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_0^3 = \left(9 - \frac{27}{2}\right) - (0 - 0)$	M1		Use limits 3 (and 0) – correct order / subtraction
		$= -4\frac{1}{2}$	A1		Obtain (-)4½
		$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_3^5 = \left(\frac{125}{3} - \frac{75}{2}\right) - \left(9 - \frac{27}{2}\right)$	M1		Use limits 5 and 3 – correct order / subtraction
		$= 8\frac{2}{3}$	A1		Obtain $8\frac{2}{3}$ (allow 8.7 or better)
		Hence total area is $13\frac{1}{6}$	A1	**7**	Obtain total area as $13\frac{1}{6}$, or exact equiv
					SR: if no longer $\int f(x)dx$, then B1 for using [0, 3] and [3, 5]
				$\boxed{8}$	
8	**(i)**	$u_4 = 10 \times 0.8^3$	M1		Attempt u_4 using ar^{n-1}
		$= 5.12$	A1	**2**	Obtain 5.12 aef
	(ii)	$S_{20} = \frac{10(1 - 0.8^{20})}{1 - 0.8}$	M1		Attempt use of correct sum formula for a GP
		$= 49.4$	A1	**2**	Obtain 49.4
	(iii)	$\frac{10}{1-0.8} - \frac{10(1 - 0.8^N)}{(1 - 0.8)} < 0.01$	M1		Attempt S_∞ using $\frac{a}{1-r}$
			A1		Obtain $S_\infty = 50$, or unsimplified equiv
		$50 - 50(1 - 0.8^N) < 0.01$	M1		Link $S_\infty - S_N$ to 0.01 and attempt to rearrange
		$0.8^N < 0.0002$ **A.G.**	A1		Show given inequality convincingly
		$\log 0.8^N < \log 0.0002$	M1		Introduce logarithms on both sides
		$N \log 0.8 < \log 0.0002$	M1		Use $\log a^b = b \log a$, and attempt to find N
		$N > 38.169$, hence $N = 39$	A1	**7**	Obtain $N = 39$ only
				$\boxed{11}$	

			Mark	Total	
9	**(i)**	$(90^o, 2), (-90^o, -2)$	B1		State at least 2 correct values
			B1	**2**	State all 4 correct values (radians is B1 B0)
	(ii)	**(a)** $180 - \alpha$	B1	**1**	State $180 - \alpha$
		(b) $-\alpha$ or $\alpha - 180$	B1	**1**	State $-\alpha$ or $\alpha - 180$ (radians or unsimplified is B1B0)
	(iii)	$2\sin x = 2 - 3\cos^2 x$			
		$2\sin x = 2 - 3(1 - \sin^2 x)$	M1		Attempt use of $\cos^2 x = 1 - \sin^2 x$
		$3\sin^2 x - 2\sin x - 1 = 0$	A1		Obtain $3\sin^2 x - 2\sin x - 1 = 0$ aef with no brackets
		$(3\sin x + 1)(\sin x - 1) = 0$	M1		Attempt to solve 3 term quadratic in $\sin x$
		$\sin x = -{}^1/_3$, $\sin x = 1$	A1		Obtain $x = -19.5^o$
		$x = -19.5^o, -161^o, 90^o$	A1√		Obtain second correct answer in range, following their x
			A1	**6**	Obtain 90^o (radians or extra answers is max 5 out of 6) SR: answer only (and no extras) is B1 B1√ B1
				$\boxed{10}$	
10	**(i)**	$(2x + 5)^4 = (2x)^4 + 4(2x)^3 5 + 6(2x)^2 5^2 + 4(2x)5^3 + 5^4$	M1*		Attempt expansion involving powers of $2x$ and 5 (at least 4 terms)
		$= 16x^4 + 160x^3 + 600x^2 + 1000x + 625$	M1*		Attempt coefficients of 1, 4, 6, 4, 1
			A1dep*		Obtain two correct terms
			A1	**4**	Obtain a fully correct expansion
	(ii)	$(2x + 5)^4 - (2x - 5)^4 = 320x^3 + 2000x$	M1		Identify relevant terms (and no others) by sign change oe
			A1	**2**	Obtain $320x^3 + 2000x$ cwo
	(iii)	$9^4 - (-1)^4 = 6560$ and $7360 - 800 = 6560$ **A.G.**	B1		Confirm root, at any point
		$320x^3 - 1680x + 800 = 0$	M1		Attempt complete division by $(x - 2)$ or equiv
		$4x^3 - 21x + 10 = 0$	A1√		Obtain quotient of $ax^2 + 2ax + k$, where a is their coeff of x^3
		$(x - 2)(4x^2 + 8x - 5) = 0$	A1		Obtain $(4x^2 + 8x - 5)$ (or multiple thereof)
		$(x - 2)(2x - 1)(2x + 5) = 0$	M1		Attempt to solve quadratic
		Hence $x = ½, x = -2½$	A1	**6**	Obtain $x = ½, x = -2½$ SR: answer only is B1 B1
				$\boxed{12}$	